January 22, 2009

PROPOSED DESIGN EQUATIONS FOR CAN/CSA S16 APPENDIX K PROVISIONS FOR STEEL MEMBERS AT HIGH TEMPERATURES

Jiro Takagi, Tokyo Metropolitan University, jtakagi@tmu.ac.jp
Gregory Deierlein, Stanford University, ggd@stanford.edu

1 INTRODUCTION

Summarized herein is a draft of proposed equations for the CSA S16-01 standard[1)](CSA S16) to calculate the axial compression and bending strengths of steel members at elevated temperatures. The proposed equations follow a similar approach by the authors to develop equations for the ANSI/AISC 360 Specification[2)] (Takagi and Deierlein[3)]) that have been balloted and approved for publication in the 2010 edition of the ANSI/ASCE 360 Specification. The equations are for bare steel (non-composite) beams, columns and beam-columns with bi-symmetric cross sections.

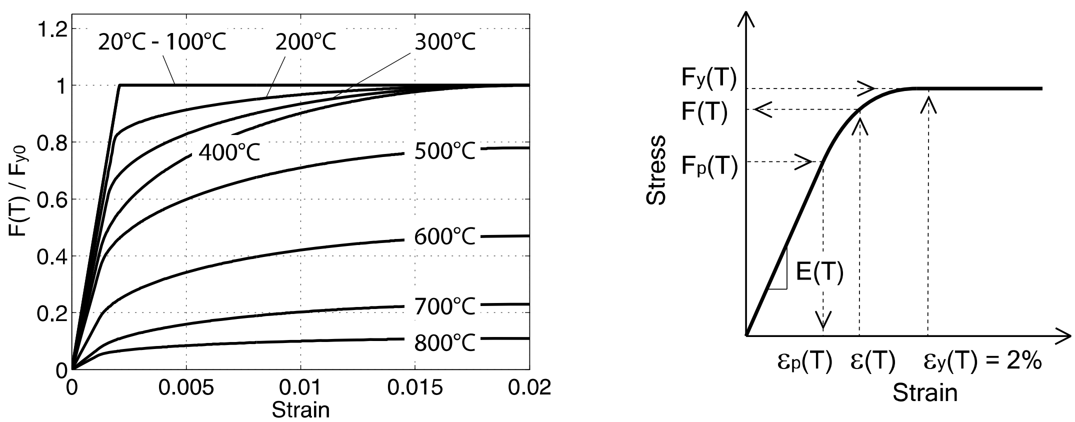
The proposed equations employ the same factors as used in the Eurocode 3[4)] (EC3) and AISC procedures to modify the elastic modulus and yield strength for high temperature (see Table 1). Since these material strength adjustment factors alone are insufficient to account for the high temperature effects on member strengths, new elevated temperature strength equations are proposed that are modeled after the current ambient temperature strength equations in CSA S16. Given the differences in behavior at elevated temperatures, the proposed high-temperature equations do not converge smoothly to the ambient temperature equations. Therefore, it is recommended that the proposed equations be limited in their application to temperatures greater than 200°C.

2 STEEL PROPERTIES AT HIGH TEMPERATURES

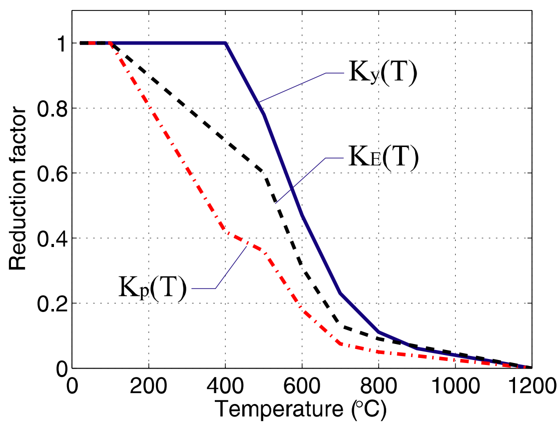
Shown in Figure 1(a) are idealized stress-strain curves for steel at elevated temperatures as specified in EC3. These stress-strain models are specified through reduction factors (see Figure 1(b)-(c)), which are defined for the proportional limit F_p, yield stress F_y, and modulus of elasticity E as follows:

$$K_p(T) = \frac{F_p(T)}{F_{p0}}, K_y(T) = \frac{F_y(T)}{F_{y0}} \text{ and } K_E(T) = \frac{E(T)}{E_0} \tag{1}$$

The terms in the denominator of Eq.(1), F_{p0}, F_{y0}, and E_0, correspond to properties at ambient temperature (20°C), and those in the numerator, $F_p(T)$, $F_y(T)$, and $E(T)$, are at the elevated temperature, T. Assuming elastic-plastic behavior at room temperature, it is assumed that F_{p0} is equal to F_{y0}. The reduction factors are summarized in Table 1 and plotted in Figure 1(c).



(a) Stress-strain curves **(b) Key parameters**



(c) Reduction factors

Figure 1 Stress-strain response at high temperatures as defined by EC3

Table 1 Stress-strain reduction factors in EC3

Temperature °C	$K_y(T)$	$K_p(T)$	$K_E(T)$
20	1.000	1.000	1.000
100	1.000	1.000	1.000
200	1.000	0.807	0.900
300	1.000	0.613	0.800
400	1.000	0.420	0.700
500	0.780	0.360	0.600
600	0.470	0.180	0.310
700	0.230	0.075	0.130
800	0.110	0.050	0.090
900	0.060	0.038	0.068
1000	0.040	0.025	0.045
1100	0.020	0.013	0.023
1200	0	0	0

3 COLUMNS

3.1 CSA S16 COLUMN STRENGTH EQUATIONS

3.1.1 CSA S16 Column Equations at Ambient Temperatures

The nominal column strength $P_{cr0,CSA}$ of the CSA S16 standard at ambient temperature is calculated as follows:

$$P_{cr0,CSA} = \left(1+\lambda^{2n}\right)^{-1/n} AF_{y0} \tag{2}$$

where n is 1.34 for hot-rolled sections and

$$\lambda = \frac{KL}{r}\sqrt{\frac{F_{y0}}{\pi^2 E_0}} = \sqrt{\frac{F_{y0}}{F_{e0}}} \tag{3}$$

3.1.2 CSA S-16 Proposed Column Strength Equations at High Temperatures

The proposed equation for CSA S16 to calculate the column strength at elevated temperatures, using a format consistent with the ambient temperature equation is as follows:

$$P_{cr,\mathrm{Pr}\,opCSA}(T) = \left(1+\lambda(T)^{2dn}\right)^{-1/dn} AF_y(T) \tag{4}$$

Where n is as given in the equation for nominal strengths (1.34 for hot-rolled sections), d is defined as 0.6, and

$$\lambda(T) = \frac{KL}{r}\sqrt{\frac{F_y(T)}{\pi^2 E(T)}} = \sqrt{\frac{F_y(T)}{F_e(T)}} \tag{5}$$

3.2 AISC COLUMN STRENGTH EQUATIONS

3.2.1 AISC Column Equations at Ambient Temperatures

The nominal column strength $P_{cr0,AISC}$ of the ANSI/AISC Specification at ambient temperature is calculated as follows:

For $F_{y0} \le 2.25F_{e0}$
$$P_{cr0,AISC} = \left[0.658^{\frac{F_{y0}}{F_{e0}}}\right] AF_{y0} \tag{6}$$

For $F_{y0} > 2.25F_{e0}$
$$P_{cr0,AISC} = 0.877AF_{e0} \tag{7}$$

where
$$F_{e0} = \frac{\pi^2 E_0}{\left(\frac{KL}{r}\right)^2} \tag{8}$$

3.2.2 AISC Column Strength Equations at High Temperatures

At elevated temperatures, the following equation has been proposed and approved for Appendix 4 to the 2010 edition of the ANSI/AISC Specification,

$$P_{cr,Prop}(T) = \left[0.42^{\sqrt{\frac{F_y(T)}{F_e(T)}}}\right] AF_y(T) \tag{9}$$

where
$$F_e(T) = \frac{\pi^2 E(T)}{\left(\frac{KL}{r}\right)^2} \quad (10)$$

and where $F_y(T)$ and $E(T)$ are calculated from the nominal values based on the adjustment factors in Table 1.

3.3 EC3 COLUMN STRENGTH EQUATIONS

3.3.1 EC3 Column Equations at Ambient Temperatures

The EC3 column strength $P_{cr0,EC3}$ at ambient temperature is calculated as follows:

$$P_{cr0,EC3} = \chi_0 P_{y0} \quad (11)$$

$$\chi_0 = \frac{1}{\varphi_0 + \sqrt{\varphi_0^2 - \bar{\lambda}_0^2}} \leq 1.0 \quad (12)$$

$$\varphi_0 = 0.5\left[1 + \alpha\left(\bar{\lambda}_0 - 0.2\right) + \bar{\lambda}_0^2\right] \quad (13)$$

$$\bar{\lambda}_0 = \sqrt{\frac{F_{y0}}{F_{e0}}} \quad (14)$$

where P_{y0} (= AF_{y0}) is the section yield strength, F_{e0} (in Equation 14) is the elastic critical load that takes into account both member and section slenderness, and α is an imperfection factor that varies from 0.13 to 0.76, depending on the member properties, such as buckling orientation, web height to flange width ratio, flange thickness, and yield strength.

3.3.2 EC3 Column Strength Equations at High Temperatures

The EC3 column strength $P_{cr,EC3}(T)$ at high temperature is calculated as follows:

$$P_{cr,EC3}(T) = \chi(T) P_y(T) \quad (15)$$

$$\chi(T) = \frac{1}{\varphi(T) + \sqrt{\varphi^2(T) - \bar{\lambda}^2(T)}} \leq 1.0 \quad (16)$$

$$\varphi(T) = 0.5\left[1 + \alpha\bar{\lambda}(T) + \bar{\lambda}^2(T)\right] \quad (17)$$

$$\bar{\lambda}(T) = \bar{\lambda}_0 \sqrt{\frac{K_y(T)}{K_E(T)}} \quad (18)$$

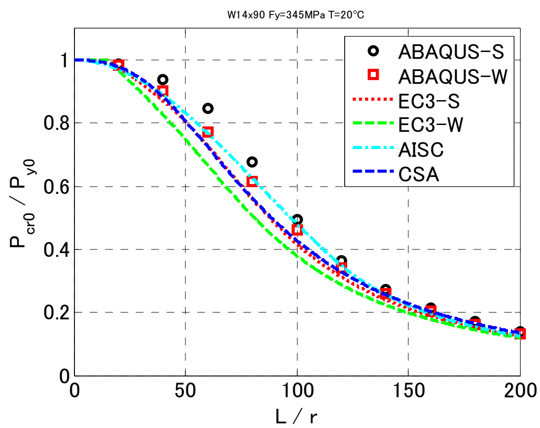
$$\alpha = 0.65\sqrt{235 / F_{y0}} \quad (19)$$

where P_y(T) is the yield strength at elevated temperatures ($=AF_y(T)$), and the other factors are as defined previously for ambient temperatures and in Table 1.

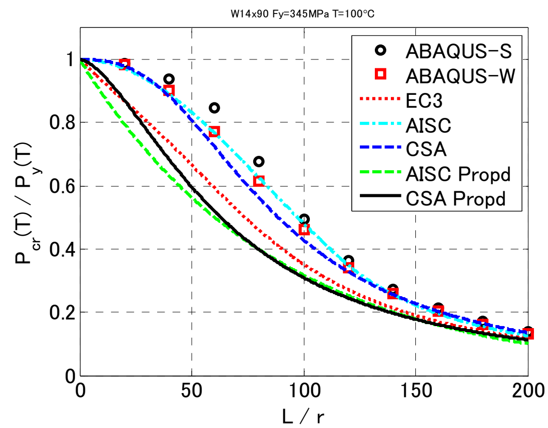
3.4 ASSESSMENT OF COLUMN STRENGTHS

The CSA S16, AISC, and EC3 column strength equations are compared to finite element method (FEM) simulations of two column sections under various temperatures and slenderness ratios in Figure 3 and Figure 4. In Figure 2, the strengths are compared for a W14x90 Gr. 50 column (W360×134, F_y=345 MPa) at temperatures ranging from ambient to 800^{o}C. In Figure 3, strengths are compared for the W14x90 Gr. 36 column (W360×134, F_y= 250 MPa) and a W14x22 Gr. 50 column (W360×32.9, F_y= 345 MPa) at 500^{o}C. Two sets of values are reported for the AISC and CSA S16. The first sets (designated "AISC" and "CSA") are based on the standard (ambient temperature) strength equations using values of $F_y(T)$ and $E(T)$. The second set (designated "AISC Propd" and "CSA Propd") are based on the newly proposed equations for high temperatures, as presented in Sections 3.1.2 and 3.2.2. The FEM simulations are performed using shell finite element models of the members under major- and minor-axis bending, using the high-temperature stress-strain curves from EC3. Further details of the FEM analyses are described in Takagi and Deierlein[3)].

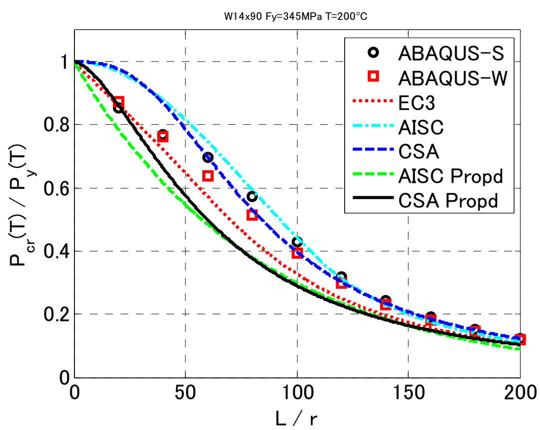
Referring to Figure 2, at ambient and 100^{o}C the standard AISC and CSA equations provide accurate estimates of the strength, as compared to the FEM solutions. At higher temperatures, strengths provided by the standard equations are quite unconservative in the inelastic buckling region, for slenderness values below L/r < 150. As reported by Takagi and Deierlein[3)], the standard AISC equations are up to 60% larger (unconservative) compare to the simulation results. Similar errors are observed for the standard CSA. For temperatures above 300^{o}C, the proposed high-temperature AISC and CSA equations reduce the errors to equal or less than those observed for the EC3 equations, all within about 20% of the FEM results.



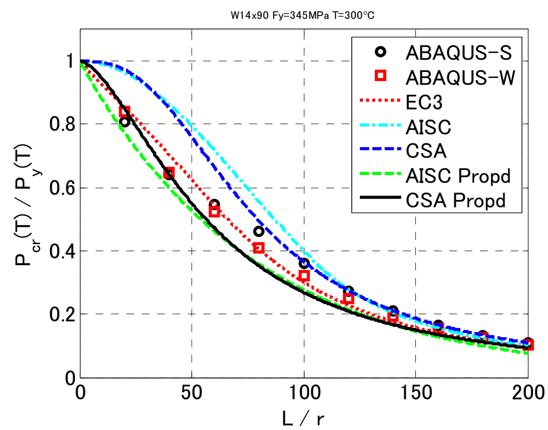
(a) ambient temperature



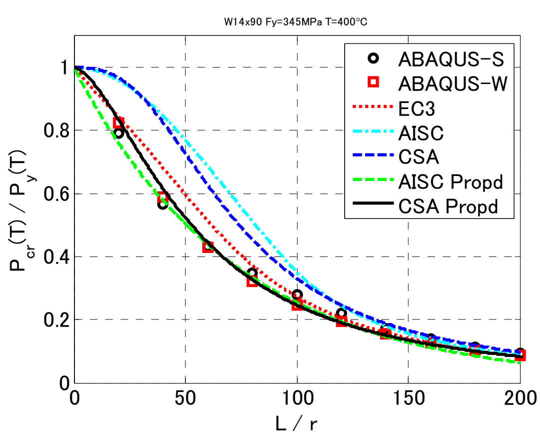
(b) 100 °C



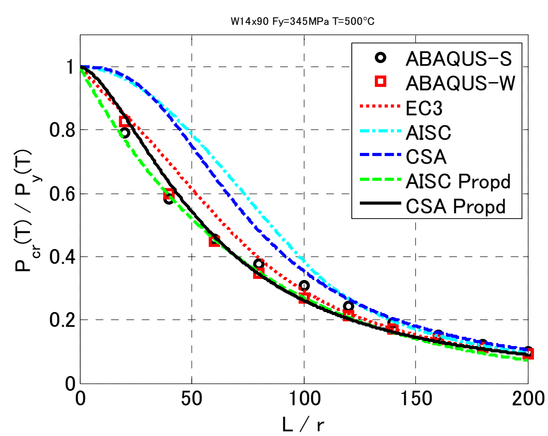
(c) 200 °C



(d) 300 °C

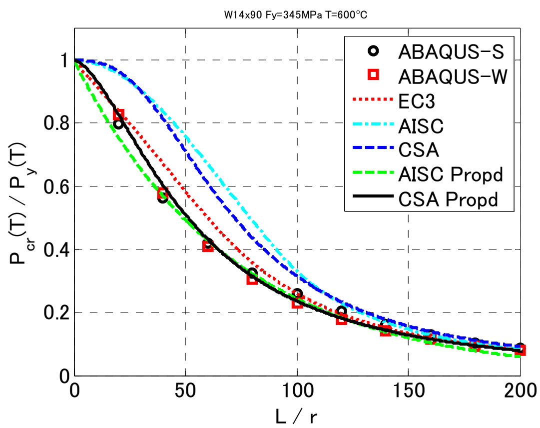


(e) 400 °C

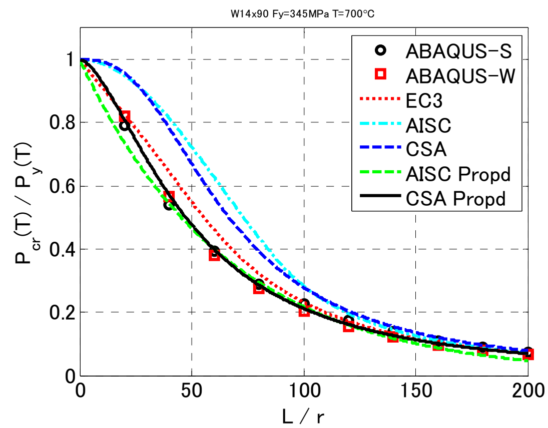


(f) 500 °C

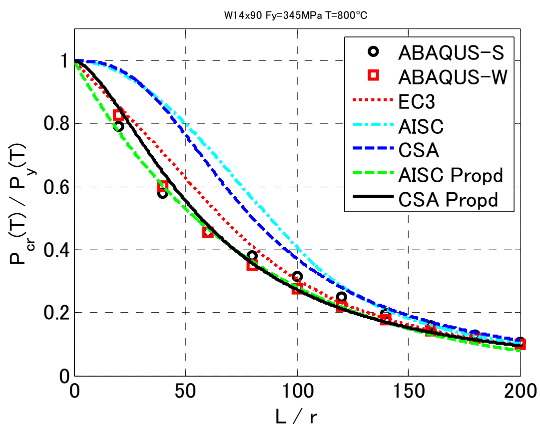
Figure 2 Critical compressive strengths of W14×90 Gr.50 column



(g) 600 °C

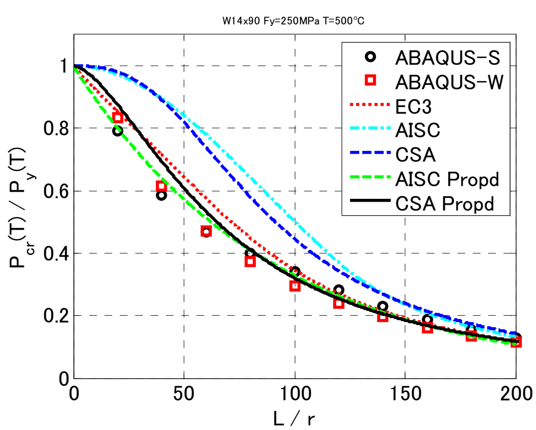


(h) 700 °C

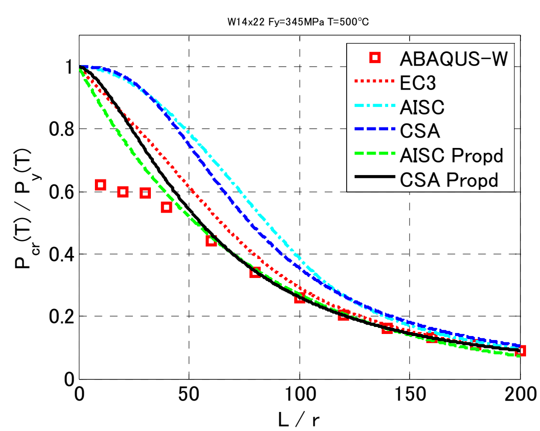


(i) 800 °C

Figure 3 Critical compressive strengths of W14×90 Gr.50 column, cont'd



(a) W14×90 Gr.36



(b) W14×22 Gr.50

Figure 4 Comparative assessment of column compression strength at 500 °C

Referring to Figure 3, the trends observed in Figure 2 are also true for the W14x90 column with lower strength steel and the W14x22 column at 500^{o}C. However, at low slenderness values (L/r < 30), the W14x22 column exhibits web local buckling, which is not captured by the column strength equations. This behavior is not unexpected since the W14x22 section has a web slenderness ratio of $h/t_w = 53$, which exceeds by a factor of about 1.5 the limit for slender webs in AISC and CSA S16. The limit for slender webs in both of these specifications is $h/tw < 670/\sqrt{F_{y0}}$ (MPa units), which for Gr. 50 (F_{y0}=345 MPa) is equal to 36. Therefore, there is not much practical significance of the local buckling behavior observed for the W14x22 column at low slenderness values.

4 BEAMS

4.1 CSA S16 Beam Strength Equations

4.1.1 CSA S16 Beam Strength Equations at Ambient Temperatures

The nominal beam strength $M_{cr0,CSA}$ of the CSA S16 standard at ambient temperature is calculated as follows:

When $M_{u0} > 0.67 M_{p0}$ $$M_{cr0,CSA} = 1.15 M_{p0}\left(1 - \frac{0.28 M_{p0}}{M_u}\right) \le M_{p0} \tag{20}$$

When $M_{u0} \le 0.67 M_{p0}$ $$M_{cr0,CSA} = M_{u0} \tag{21}$$

where

$$M_{u0} = \frac{\omega_2 \pi}{L}\sqrt{E_0 I_y G_0 J + I_y C_w \left(\frac{\pi E_0}{L}\right)^2} \tag{22}$$

where M_{po} is the plastic moment capacity (based on F_{y0}) and ω_2 is the moment gradient coefficient (equal to 1.0, when the moment at any point within the unbraced length is larger than the maximum end moment).

4.1.2 CSA S16 Proposed Beam Strength Equations at High Temperatures

The proposed nominal beam strength $M_{cr,\Pr opCSA}(T)$ for the CSA S16 standard at high temperatures is calculated as follows:

$$M_{cr,\Pr opCSA}(T) = C_K M_p(T) + (1 - C_K) M_p(T) \left(1 - \left(\frac{C_K M_p(T)}{M_u(T)} \right)^{0.5} \right)^{Cz(T)} \tag{21}$$

where C_K is 0.12, $M_p(T)$ is the plastic moment calculated using $F_y(T)$, $M_u(T)$ is the elastic critical load at elevated temperatures, calculated using Equation 22 by substituting $E(T)$ and $G(T)$ for E_0 and G_0, and

$$C_Z(T) = \frac{T + 800}{500} \leq 2.4 \tag{22}$$

where T is the elevated temperature (in ^{o}C). Although not shown in Equation 23, the same moment gradient term, ω_2 , originally developed for ambient temperatures, could be applied at elevated temperatures.

4.2 AISC Beam Strength Equations

4.2.1 AISC Beam Strength Equations at Ambient Temperatures

The nominal beam strength $M_{cr0,AISC}$ of the ASCE Specification at ambient temperature is calculated as follows:

When $L_b \leq L_{p0}$
$$M_{cr0,AISC} = M_{p0} \tag{23}$$

When $L_{p0} < L_b \leq L_{r0}$

$$M_{cr0,AISC} = C_b \left[M_{p0} - (M_{p0} - M_{r0}) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_{p0} \tag{24}$$

When $L_{r0} < L_b$
$$M_{cr0,AISC} = F_{cr0} S_x \leq M_{p0} \tag{25}$$

where M_{p0} is the plastic moment strength and

$$M_{r0} = S_x F_{L0} \tag{26}$$

$$F_{L0} = 0.7 F_{y0} \tag{27}$$

$$L_{p0} = 1.76 r_y \sqrt{\frac{E_0}{F_{y0}}} \tag{28}$$

$$F_{cr0} = \frac{C_b \pi^2 E_0}{\left(\dfrac{L_b}{r_{ts}}\right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_o} \left(\frac{L_b}{r_{ts}}\right)^2} \tag{29}$$

$$L_{r0} = 1.95 r_{ts} \frac{E_0}{0.7 F_{y0}} \sqrt{\frac{Jc}{S_x h_o}} \sqrt{1 + \sqrt{1 + 6.76 \left(\frac{0.7 F_{y0}}{E_0} \frac{S_x h_o}{Jc}\right)^2}} \tag{30}$$

$$r_{ts}^2 = \frac{\sqrt{I_y C_w}}{S_x} \tag{31}$$

The coefficients c and C_b are both 1.0 for a doubly symmetric I-shape section subject to uniform bending moment. Note that Equation 31, which follows the format of the forthcoming 2010 AISC Specification, is essentially the same elastic critical load as given by Equation 22 for the CSA S16 standard.

4.2.2 AISC Proposed Beam Strength Equations at High Temperatures

The nominal beam strength $M_{cr0,AISC}$ of the ASCE Specification at elevated temperatures is calculated as follows:

When $L_b \le L_r(T)$

$$M_{cr,AISC}(T) = C_b \left[M_r(T) + \left(M_p(T) - M_r(T)\right)\left(1 - \frac{L_b}{L_r(T)}\right)^{C_X(T)} \right] \tag{32}$$

When $L_r(T) < L_b$ $\quad M_{cr,AISC}(T) = F_{cr}(T) S_x \quad (33)$

where

$$C_X(T) = 0.6 + \frac{T}{250} \leq 3.0 \text{ (T in }^{o}\text{C)} \tag{34}$$

$$M_p(T) = Z_x F_y(T) \tag{35}$$

$$M_r(T) = S_x F_L(T) \tag{36}$$

$$F_L(T) = \left(K_p(T) - 0.3K_y(T)\right)F_{y0} \tag{37}$$

and $F_{cr}(T)$ is calculated using Equation 31 by substituting $E(T)$ for E_0 and $L_r(T)$ is calculated using Equation 32 by substituting $E(T)$ for E_0, $F_L(T)$ for $0.7F_{y0}$.

4.3 EC3 BEAM STRENGTH EQUATIONS

4.3.1 EC3 Beam Strength Equations at Ambient Temperatures

The nominal beam strength $M_{cr0,EC3}$ of the EC3 provisions at ambient temperatures is calculated as follows:

$$M_{cr0,EC3} = \chi_{LT0} M_{p0} \tag{38}$$

$$\chi_{LT0} = \frac{1}{\varphi_{LT0} + \sqrt{\varphi_{LT0}^2 - \bar{\lambda}_{LT0}^2}} \leq 1.0 \tag{39}$$

where

$$\varphi_{LT0} = 0.5\left[1 + \alpha_{LT}(\bar{\lambda}_{LT0} - 0.2) + \bar{\lambda}_{LT0}^2\right] \tag{40}$$

$$\bar{\lambda}_{LT0} = \sqrt{\frac{M_{p0}}{M_{cr0,e}}} \tag{41}$$

$$M_{p0} = Z_x F_{y0} \tag{42}$$

and χ_{LT0} is the reduction factor for lateral torsional buckling, α_{LT} is an imperfection factor which depends on the section proportions (α_{LT} = 0.21 is used for rolled sections with the web height to flange width ratio $h/b_f \leq 2$ and α_{LT} = 0.34 for $h/b_f > 2$).

4.3.2 EC3 Beam Strength Equations at High Temperatures

The nominal beam strength $M_{cr0,EC3}$ of the EC3 provisions at high temperatures is calculated using Equations 40 and 41, but with the following substitutions:

$$\varphi_{LT}(T)=0.5[1+\alpha_{LT}\bar{\lambda}_{LT}(T)+\bar{\lambda}_{LT}^2(T)] \tag{45}$$

$$\alpha_{LT}=0.65\sqrt{235/F_{y0}} \quad (F_{y0} \text{ in MPa units}) \tag{46}$$

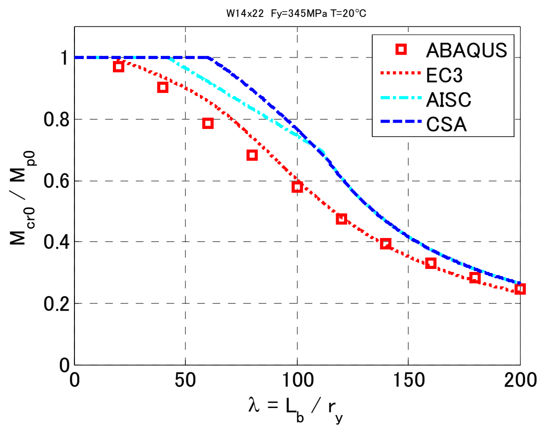
$$\bar{\lambda}_{LT}(T)=\bar{\lambda}_{LT0}\sqrt{\frac{K_y(T)}{K_E(T)}} \tag{47}$$

$$M_p(T) = Z_x F_y(T) \tag{48}$$

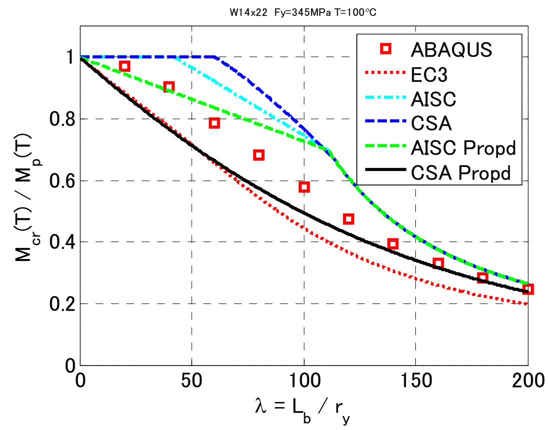
4.4 ASSESSMENT OF BEAM STRENGTHS

The CSA S16, AISC, and EC3 beam strength equations are compared to finite element method (FEM) simulations of two column sections under various temperatures and slenderness ratios in Figure 3 and Figure 4. In Figure 4, the strengths are compared for a W14x22 Gr. 50 beam (W360×32.9, F_y=345 MPa) at temperatures ranging from ambient to 800°C. In Figure 5, strengths are compared for the W14x22 Gr. 36 beam (W360×32.9, F_y = 250 MPa) and a W14x90 Gr. 50 beam (W360×134, F_y = 345 MPa) at 500°C. Two sets of values are reported for the AISC and CSA S16. The first sets (designated "AISC" and "CSA") are based on the standard (ambient temperature) strength equations using values of $F_y(T)$ and $E(T)$. The second set (designated "AISC Propd" and "CSA Propd") are based on the newly proposed equations for high temperatures, as presented in Sections 4.1.2 and 4.2.2. The FEM simulations are performed using shell finite element models of the members under major- and minor-axis bending, using the high-temperature stress-strain curves from EC3. Further details of the FEM analyses are described in Takagi and Deierlein[3)].

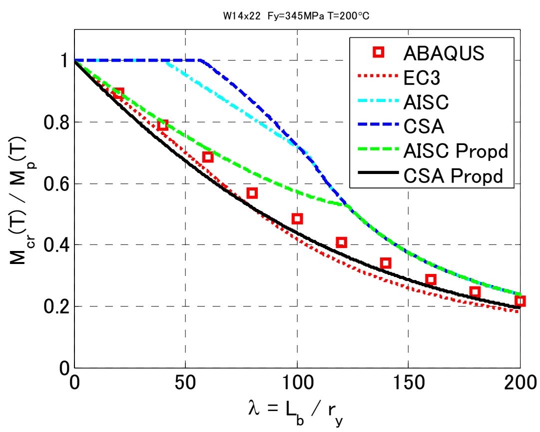
Referring to Figure 4a, at ambient temperatures, the standard AISC and CSA equations are somewhat unconservative, whereas the EC3 equations provide accurate estimates of the strength, as compared to the FEM solutions. At higher temperatures (Figures 4b-i), strengths provided by the standard AISC and CSA equations are quite unconservative in the inelastic buckling region. As reported by Takagi and Deierlein[3)], the standard AISC equations are up to 80% larger (unconservative) compared to the simulation results. Similar errors are observed for the standard CSA. For temperatures above



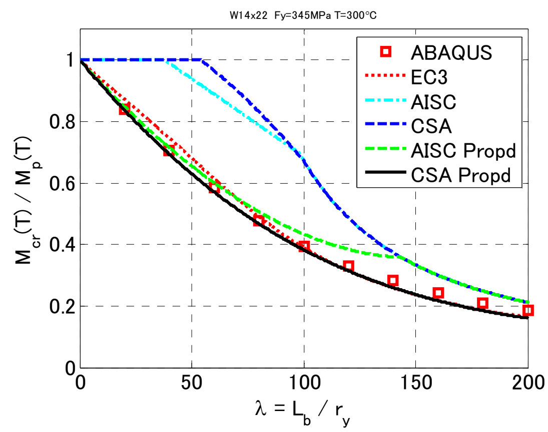
(a) ambient temperature



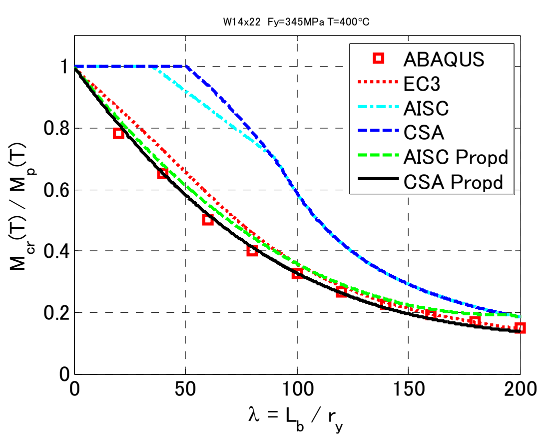
(b) 100 °C



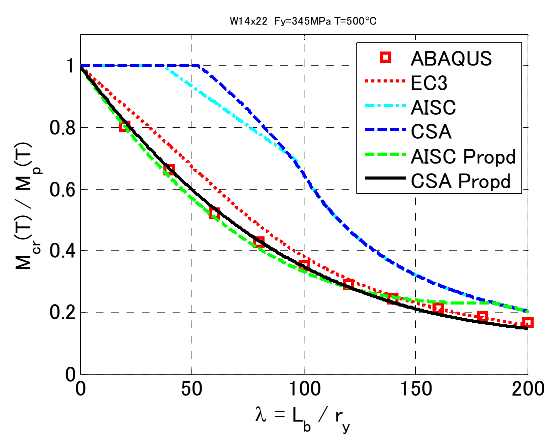
(c) 200 °C



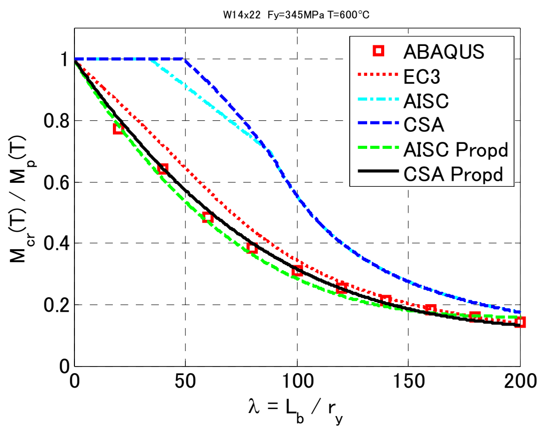
(d) 300 °C



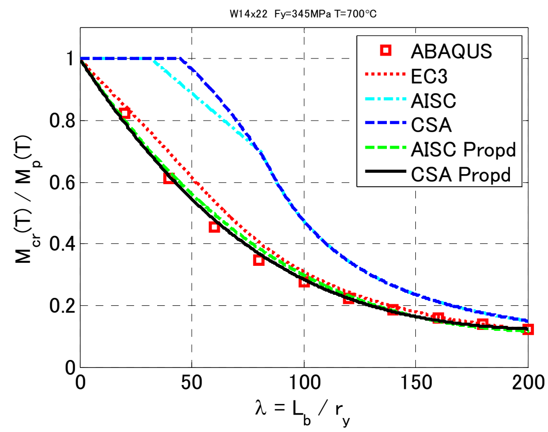
(e) 400 °C



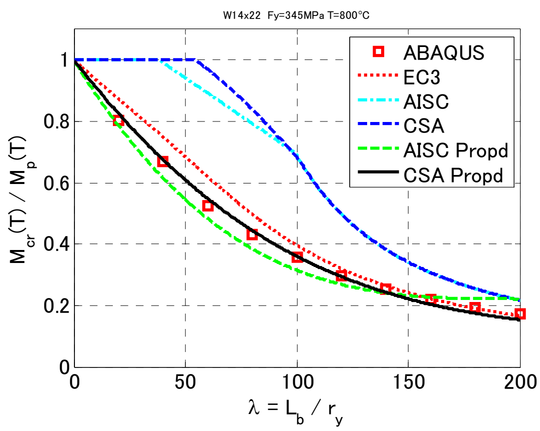
(f) 500 °C



(g) 600 °C

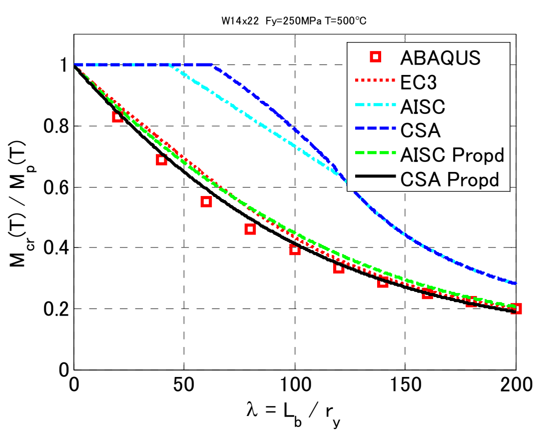


(h) 700 °C

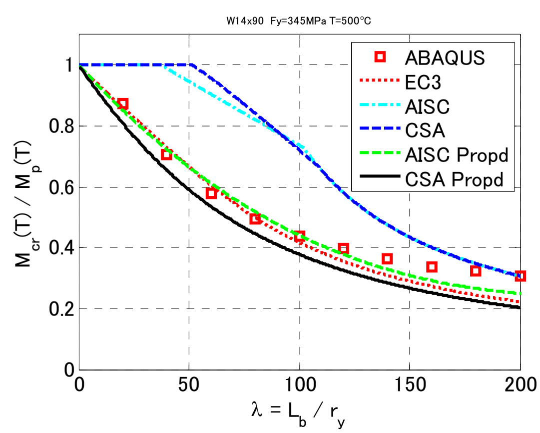


(i) 800 °C

Figure 5 Critical bending moment strengths of W14×22 Gr.50 beam



(a) W14×22 Gr.36



(b) W14×90 Gr.50

Figure 6 Comparative assessment of beam bending moment strength at 500 °C

200^{o}C, the proposed high-temperature AISC and CSA equations reduce the errors to equal or less than those observed for the EC3 equations, all within about 20% of the FEM results. Referring to Figure 5, the proposed AISC and CSA equations provide accurate results at 500^{o}C for the Gr. 36 material and the other beam section.

5 COMPARISON OF THE SECTION CLASSIFICATION CRITERIA

Takagi and Deierlein[3)] considered the influence of local buckling in their study of the AISC design requirements and generally concluded that the cross section compactness criteria, based on ambient temperatures, can apply at elevated temperatures as well. Comparing Table 2 (CSA S16 compactness criteria) and

Table 3 (AISC criteria), it can be seen that the maximum width to thickness ratios are generally more strict in CSA than AISC. Therefore, it follows that the conclusions reached by Takagi and Deierlein for the AISC criteria apply to CSA S16 as well, i.e., that the section designations based on properties at ambient temperature, can apply at elevated temperatures as well. Of course, the class definitions of CSA S16 would need to be modified to reflect plastic and yield strengths at high temperatures. For example,

- Class 1 sections permit attainment of the plastic moment, *Mp(T),* and subsequent redistribution of the bending moment;
- Class 2 sections permit attainment of the plastic moment, *Mp(T),* but need not allow for subsequent moment distribution;
- Class 3 sections permit attainment of the yield moment, *My(T),* ; and
- Class 4 sections generally have local buckling of elements in compression as the limit state of structural resistance.

Table 2 Section Compactness Criteria in CSA S16

	Compression	Flexural Compression (Class 1) *1)	Flexural Compression (Class 2) *1)	Flexural Compression (Class 3) *1)
Flange ($\frac{b_f}{2t_f}$)	$\frac{200}{\sqrt{F_y}}$	$\frac{145}{\sqrt{F_y}}$	$\frac{170}{\sqrt{F_y}}$	$\frac{200}{\sqrt{F_y}}$
Web ($\frac{h}{t_w}$)	$\frac{670}{\sqrt{F_y}}$	$\frac{1100}{\sqrt{F_y}}\left(1-0.39\frac{C_f}{\phi C_y}\right)$ [$\frac{671}{\sqrt{F_y}}$] *2)	$\frac{1700}{\sqrt{F_y}}\left(1-0.61\frac{C_f}{\phi C_y}\right)$ [$\frac{663}{\sqrt{F_y}}$] *2)	$\frac{1900}{\sqrt{F_y}}\left(1-0.65\frac{C_f}{\phi C_y}\right)$ [$\frac{665}{\sqrt{F_y}}$] *2)

Notes:
*1) ϕ, C_f, and C_y are resistance factor, factored axial load, and axial compressive load at yield stress, respectively, and F_y is in *MPa* units.
*2) : the maximum width to thickness ratios assuming that C_f equals to C_y multiplied by ϕ.

Table 3 Section Compactness Criteria in AISC

	Compression *1)	Flexure	
	λ_r	λ_p	λ_r
Flange ($\frac{b_f}{2t_f}$)	$0.56\sqrt{\frac{E}{F_y}}$ [$\frac{250}{\sqrt{F_y}}$] *1)	$0.38\sqrt{\frac{E}{F_y}}$ [$\frac{170}{\sqrt{F_y}}$] *1)	$0.83\sqrt{\frac{E}{F_y}}$ [$\frac{372}{\sqrt{F_y-10}}$] *1)
Web ($\frac{h}{t_w}$)	$1.49\sqrt{\frac{E}{F_y}}$ [$\frac{666}{\sqrt{F_y}}$] *1)	$3.76\sqrt{\frac{E}{F_y}}$ [$\frac{1682}{\sqrt{F_y}}$] *1)	$5.70\sqrt{\frac{E}{F_y}}$ [$\frac{2549}{\sqrt{F_y}}$] *1)

Notes:
*1) Fy is in *MPa* units.

*2) Steel sections are classified in Compact ($\lambda < \lambda_p$), Non-compact ($\lambda_p < \lambda < \lambda_r$), and Slender ($\lambda > \lambda_r$), in AISC with respect to their width to thickness ratios.

Reference

1) CAN/CSA-S16-01: Limit States Design of Steel Structures, A National Standard of Canada, reprinted August 2005.
2) American Institute of Steel Construction, Inc. (AISC 2009), Specification for structural steel buildings, ANSI/AISC 360-2010, Chicago, IL; 2010 (latest ballot version, January 2009)
3) Takagi J, Deierlein GG: Strength Design Criteria for Steel Members at Elevated Temperatures, Journal of Constructional Steel Research (63), pp1036-1050, 2007.
4) European Committee for Standardisation (CEN): Eurocode 3. Design of Steel Structures – part 1-2. General rules – Structural fire design. Draft prEN 1993-1-2, Stage 49 Draft, Brussels, Belgium; 2003.

SYMBOLS

X_0 and $X(T)$ indicate that X is a function of temperature.

X_0 is a property at ambient temperature and $X(T)$ is that at elevated temperature T.

X_T is also used, if the value of X at elevated temperatures is different from that at ambient temperature and X is constant at any elevated temperature.

A	=	Cross-sectional area (m^2)
A_B	=	Cross-sectional area of unthreaded part of bolt (m^2)
A_f	=	Cross-sectional area of lumped beam section (half of cross-sectional area of beam) (m^2)
C_w	=	Warping constant (m^5)
C_X	=	Exponent in equation for proposed critical moment
DL	=	Dead load
D_B	=	Diameter of bolt (m)
E	=	Modulus of elasticity (N/m^2)
E_t	=	Tangent stiffness (N/m^2)
$E_{t,ave}$	=	Average tangent stiffness in a section (N/m^2)
F_{Bv}	=	Ultimate shearing stress of bolt (N/m^2)

F_L	=	Initial yield stress (N/m^2)
F_e	=	Elastic buckling stress (N/m^2)
F_p	=	Stress at the proportional limit (N/m^2)
F_r	=	Residual stress (N/m^2)
F_y	=	Yield stress (N/m^2)
$F_{y,char}$	=	Characteristic 0.2 % off-set yield strength (N/m^2)
G	=	Shear modulus of elasticity (N/m^2)
I_x , I_y	=	Moment of inertia about strong and weak axis (m^4)
J	=	Torsional constant (m^4)
K	=	Effective buckling length factor
K_p , K_y , K_E	=	Reduction factors for the proportional limit, yield stress, and modulus of elasticity respectively
K_s	=	Longitudinal constraint spring stiffness for beams (N/m)
K_{yB}	=	Reduction factor for bolt shear strength
L	=	Length (m)
L_b	=	Unbraced length for beams (m)
LL	=	Live load
M_p	=	Plastic moment (Nm)
M_r	=	Initial yield moment (Nm)
$M_{cr,AISC}$, ($M_{crx,AISC}$)	=	Nominal moment (about strong axis) in AISC (Nm)
$M_{cr,EC3}$, ($M_{crx,EC3}$)	=	Nominal moment (about strong axis) in EC3 (Nm)
$M_{cr,Prop}$, ($M_{crx,Prop}$)	=	Proposed nominal moment (about strong axis) (Nm)
$M_{cr,e}$	=	Elastic critical moment (Nm)
$M_{cr,tan}$	=	Nominal moment by tangent modulus theory (Nm)
M_{ux}	=	Factored bending moment about strong axis (Nm)
$M_{x,end}$	=	Bending moment about strong axis at the ends (Nm)
N_B	=	Number of bolt at connection
P_B	=	Peak strength of longitudinal spring for bolted connections (N)

P_c	=	Vertical load carrying capacity of column (N)
$P_{cr,AISC}$, ($P_{cry,AISC}$)	=	Nominal axial strength of column (for flexural buckling about weak axis) in AISC (N)
$P_{cr,EC3}$, ($P_{cry,EC3}$)	=	Nominal axial strength of columns (for flexural buckling about weak axis) in EC3 (N)
$P_{cr,Prop}$, ($P_{cry,Prop}$)	=	Proposed nominal axial strength of column (for flexural buckling about weak axis) (N)
$P_{cr,tan}$	=	Nominal strength by tangent modulus theory (N)
P_g	=	Gravity load (N)
P_n	=	Nominal axial strength of column in AISC (N)
P_s	=	Resistance force of vertical spring for interior column (N)
P_u	=	Factored axial load (N)
Q_u	=	Factored load
R_{Bn}	=	Nominal shear strength of bolt (N)
R_n	=	Nominal resistance in AISC
R_{Btest}	=	Experimental shear strength of bolt (N)
S	=	Elastic section modulus (m^3)
S_x	=	Elastic section modulus about strong axis (m^3)
Z	=	Plastic section modulus (m^3)
Z_x	=	Plastic section modulus about strong axis (m^3)
T	=	Temperature (°C)
T_L	=	Temperature at lower flange of beam (°C)
T_{cr}	=	Critical temperature (°C)
T_{smax}	=	Maximum temperature of steel in fire simulation (°C)
dT	=	Increment of temperature (= T - 20 °C) (°C)
$\mathbf{b}_{LB}$	=	Critical imperfection mode for local buckling (m)
b_f	=	Flange width of section (m)
c_p	=	Specific heat (J/kg°C)
g	=	Limit-state function
h	=	Height of section (m)

h_c	=	Convection coefficient (W/m^2°C)
h_{net}	=	Height of lumped beam section (m)
k	=	Thermal conductivity (W/m°C)
r	=	Governing radius of gyration (m)
r_x , r_y	=	Radius of gyration about strong and weak axis (m)
t	=	Time (sec)
t_f , t_w	=	Flange and web thickness of section respectively (m)
(x, y)	=	Coordinate in sections (m)
Δ	=	Vertical displacement of column (m)
Δ_B	=	Deformation capacity of longitudinal spring for bolted connections (m)
Δ_{Bp} , Δ_{Be}	=	Relative displacement at longitudinal spring for bolted connections at peak strength, and vanishing strength (m)
α	=	Imperfection factor for flexural buckling in EC3, Thermal expansion coefficient
α_{LT}	=	Imperfection factor for lateral-torsional buckling in EC3
α_x , α_y	=	Imperfection factor for flexural buckling about strong and weak axis in EC3
δ	=	Displacement (m), Coefficient of variation
δ_{F_y}	=	Coefficient of variation of steel strength at 2 % strain
$\delta_{F_y,PS}$	=	Coefficient of variation of 0.2 % off-set steel yield strength
$\delta_{F_yT\|PS}$	=	Coefficient of variation of steel strength at 2 % strain at elevated temperatures with respect to 0.2 % off-set yield strength
δ_{yB}	=	Coefficient of variation of shear strength of bolt
ε	=	Strain, Emissivity
ϕ	=	Curvature in beam section (1/m), Resistance factor in AISC
λ	=	Slenderness ratio in AISC
$\bar{\lambda}$	=	Slenderness ratio for flexural buckling in EC3
$\bar{\lambda}_{LT}$	=	Slenderness ratio for lateral torsional buckling in EC3
λ_p	=	Slenderness ratio for transition between full plastic bending and

		inelastic lateral-torsional buckling in AISC
λ_r	=	Slenderness ratio for transition between inelastic and elastic lateral-torsional buckling in AISC
λ_{rf} , λ_{rw}	=	Limiting width to thickness ratio for local buckling of flange and web in AISC
ρ	=	Density (kg/m^3)
σ	=	Stress (N/m^2), Standard deviation, the Stefan-Boltzmann constant (5.67×10^{-8} W/m^2°C^4)
σ_a	=	Generated stress by applied load (N/m^2)
σ_r	=	Residual stress (N/m^2)
σ_y	=	Yield stress (N/m^2)
μ	=	Mean
μ_{F_y}	=	Mean of steel strength at 2 % strain (N/m^2)
$\mu_{F_y,PS}$	=	Mean of 0.2 % off-set steel yield strength (N/m^2)
$\mu_{F_y\|PS}(T)$	=	Mean of Ratio between 2 % strength at elevated temperatures and 0.2 % off-set strength
μ_{Ks}	=	Mean of longitudinal spring stiffness for beams by surrounding floor framing (N/m)
μ_{yB}	=	Mean of shear strength of bolt (N)
μ_{yBtest}	=	Mean of tested shear strength of bolt (N)
χ	=	Reduction factor for flexural buckling in EC3
χ_{LT}	=	Reduction factor for lateral torsional buckling in EC3